Error Bounds for Linear Complementarity Problems of S-Nekrasov Matrices and B-S-Nekrasov Matrices

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Abstract. First, the estimation of the error bounds of the S-Nekrasov matrix is studied. Using the properties of S-Nekrasov matrices $\boldsymbol{M}$ and $\tilde{\boldsymbol{M}} = \boldsymbol{I} - \boldsymbol{D} + \boldsymbol{DM}$, The properties of inequalities, And the estimators of the upper bound of infinite norm for the inverse matrix of $\boldsymbol{M}$ matrix, The estimation of the error bounds of the matrix is obtained, Further on this basis the estimation formula for the error bounds of the B-S-Nekrasov matrix is also obtained.

1 Introduction

The Linear complementarity problem (Lcp$(\boldsymbol{M},\boldsymbol{q})$)is to find a vector $\boldsymbol{x} \in \mathbf{R}^n$， such that

$$\boldsymbol{x} \ge 0, \boldsymbol{Mx} + \boldsymbol{q} \ge 0, \left(\boldsymbol{Mx} + \boldsymbol{q}\right)^T x = 0$$

Where $\boldsymbol{M} = \left(m_{ij}\right) \in \mathbf{R}^{n \times n}, \boldsymbol{q} \in \mathbf{R}^n$

Linear complementarity problem (Lcp$(\boldsymbol{M},\boldsymbol{q})$)has various applications in the Nash equilibrium point of a bimatrix game, the network equilibrium problem, the contact problem and the free boundary problem for journal bearing, for details, see[1-5].

It is well-known that the Lcp$(\boldsymbol{M},\boldsymbol{q})$ has a unique solution for any $\boldsymbol{q} \in \mathbf{R}^n$ if and only if $\boldsymbol{M}$ is a $\boldsymbol{P}$ matrix.

Here, a matrix $\boldsymbol{M} = \left(m_{ij}\right) \in \mathbf{R}^{n \times n}$ is called a $\boldsymbol{P}$ matrix if all its principal minors are positive. At this point, the problem not only has a unique solution, but also can easily get the error boundary [6].

Such as, Chen gave the following error bound of the Lcp$(\boldsymbol{M},\boldsymbol{q})$ When $\boldsymbol{M}$ is a $\boldsymbol{P}$ matrix in References [7] $\|x - x^*\|_\infty \le \max_{d \in [0,1]^n} \|(\boldsymbol{I} - \boldsymbol{D} + \boldsymbol{DM})^{-1}\|_\infty \|r(x)\|_\infty$

$r(x) = \min\{x, \boldsymbol{M}x + \boldsymbol{q}\}$， $\boldsymbol{D} = diag(d_i)$， $\boldsymbol{d} = [d_1, d_2, \cdots, d_n]^T (0 \le d_i \le 1)$, In this error boundary problem, The hardest thing to find $\max_{d \in [0,1]^n} \|(\boldsymbol{I} - \boldsymbol{D} + \boldsymbol{DM})^{-1}\|_\infty$. About this difficulty, Many scholars in References$[8-15]$, When the matrix for the Lcp$(\boldsymbol{M},\boldsymbol{q})$ belongs to $\boldsymbol{P}$ matrix or some subclass of $\boldsymbol{P}$ matrix, a lot of research has been done.

This paper studies the error bounds for Linear complementarity problems of S-Nekrasov matrices and B-S-Nekrasov matrices for the new subclass of $\boldsymbol{P}$ matrix.

Let $\boldsymbol{M} = (m_{ij}) \in C^{n,n}$， if $|m_{ii}| > h_i^S(\boldsymbol{M})$， $i \in S$， $\left(|m_{ii}| - h_i^S(\boldsymbol{M})\right)\left(|m_{jj}| - h_j^{\bar{S}}(\boldsymbol{M})\right) > h_i^{\bar{S}}(\boldsymbol{M}) h_j^S(\boldsymbol{M})$， $i \in S$， $i \in \bar{S}$， then $\boldsymbol{M}$ is $S-Nekrasov$ matrix.

where， $S \subseteq N$， $S \ne \varnothing$， $r_i(\boldsymbol{M}) = \sum_{j \ne i}^{n} |m_{ij}|$， $r_i^S(\boldsymbol{M}) = \sum_{j \in S \setminus \{i\}}^{n} |m_{ij}|$， $h_1^S(\boldsymbol{M}) = r_1^S(\boldsymbol{M})$，

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$$h_i^S(\boldsymbol{M})=\sum_{j=1}^{i-1}\frac{|m_{ij}|}{|m_{jj}|}h_j^S(\boldsymbol{M})+\sum_{j=i+1,j\in S}^{n}|m_{ij}|,\quad z_1(A)=1,\quad z_i(\boldsymbol{M})=\sum_{j=1}^{i-1}|m_{ij}|\frac{z_j(\boldsymbol{M})}{|m_{jj}|}+1.$$

Lemma 1[16] Let $\boldsymbol{M}=(m_{ij})\in C^{n,n}$ be S-Nekrasov matrix, $S\subseteq N$, $S\neq\varnothing$, $m_{ii}>0$, let $\tilde{\boldsymbol{M}}=\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M}=(\tilde{m}_{ij})$, $\boldsymbol{D}=diag(d_i)$, $0\le d_i\le 1$, then $\forall i,j\in N$

$$d_i h_i^S(\boldsymbol{M})\ge h_i^{\bar{S}}(\tilde{\boldsymbol{M}}),\qquad d_j h_j^S(\boldsymbol{M})\ge h_j^{\bar{S}}(\tilde{\boldsymbol{M}})$$

when $i\in S, j\in\bar{S}$, $\bar{S}=N\setminus S$, then $(\tilde{m}_{ii}-h_i^S(\tilde{\boldsymbol{M}}))(\tilde{m}_{jj}-h_j^{\bar{S}}(\tilde{\boldsymbol{M}}))>h_i^{\bar{S}}(\tilde{\boldsymbol{M}})h_j^S(\tilde{\boldsymbol{M}})$.

Lemma 2[16] Let $\boldsymbol{M}=(m_{ij})\in C^{n,n}$ be S-Nekrasov matrix, $S\subseteq N$, $S\neq\varnothing$, $m_{ii}>0$, let $\tilde{\boldsymbol{M}}=\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M}=(\tilde{m}_{ij})$, $\boldsymbol{D}=diag(d_i)$, $0\le d_i\le 1$, then $\tilde{\boldsymbol{M}}$ is S-Nekrasov matrix, if $\forall i\in N$, then $\frac{h_i^S(\tilde{\boldsymbol{M}})}{\tilde{m}_{ii}}\le\frac{h_i^S(\boldsymbol{M})}{m_{ii}}$, if $\forall j\in N$, then $\frac{h_j^{\bar{S}}(\tilde{\boldsymbol{M}})}{\tilde{m}_{jj}}\le\frac{h_j^{\bar{S}}(\boldsymbol{M})}{m_{jj}}$.

Lemma 3[15] Let $\gamma>0$, $\eta\ge 0$, Then for any $\forall x\in[0,1]$, $\frac{1}{1-x+\gamma x}\le\frac{1}{\min\{\gamma,1\}}$ and $\frac{\eta x}{1-x+\gamma x}\le\frac{\eta}{\gamma}$.

Lemma 4[15] Let $\boldsymbol{M}=(m_{ij})\in C^{n,n}$ be a Nekrasov matrix with $m_{ii}>0$, let $\tilde{\boldsymbol{M}}=\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M}=(\tilde{m}_{ij})$, $\boldsymbol{D}=diag(d_i)$, $0\le d_i\le 1$. Then $z_i(\tilde{\boldsymbol{M}})\le\eta_i(\boldsymbol{M})$, $\frac{z_i(\tilde{\boldsymbol{M}})}{\tilde{m}_{ii}}\le\frac{\eta_i(\boldsymbol{M})}{\min\{m_{ii},1\}}$, where $z_1(\tilde{\boldsymbol{M}})=\eta_1(\boldsymbol{M})=1$, $z_i(\tilde{\boldsymbol{M}})=\sum_{j=1}^{i-1}\frac{|\tilde{m}_{ij}|}{|\tilde{m}_{jj}|}z_j(\tilde{\boldsymbol{M}})+1$, $\eta_i(\boldsymbol{M})=\sum_{j=1}^{i-1}\frac{|m_{ij}|}{\min\{|m_{jj}|,1\}}\eta_j(\boldsymbol{M})+1$.

Lemma 5[17] Let $\boldsymbol{M}=(m_{ij})\in C^{n,n}$ be S-Nekrasov matrix, then

$$\|\boldsymbol{M}^{-1}\|_\infty\le\max_{i\in N}z_i(\boldsymbol{M})\max_{i\in S,j\in\bar{S}}\max\left\{\chi_{ij}^S(\boldsymbol{M}),\chi_{ji}^{\bar{S}}(\boldsymbol{M})\right\}\tag{1}$$

and $$\|\boldsymbol{M}^{-1}\|_\infty\le\max_{i\in N}\frac{z_i(\boldsymbol{M})}{|m_{ii}|}\max_{i\in S,j\in\bar{S}}\max\left\{\tilde{\chi}_{ij}^S(\boldsymbol{M}),\tilde{\chi}_{ji}^{\bar{S}}(\boldsymbol{M})\right\}\tag{2}$$

Where $$\chi_{ij}^S(\boldsymbol{M})=\frac{|m_{ii}|-h_i^S(\boldsymbol{M})+h_j^S(\boldsymbol{M})}{(|m_{ii}|-h_i^S(\boldsymbol{M}))(|m_{jj}|-h_j^{\bar{S}}(\boldsymbol{M}))-h_i^{\bar{S}}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

$$\tilde{\chi}_{ij}^S(\boldsymbol{M})=\frac{|m_{ii}||m_{jj}|-|m_{jj}|h_i^S(\boldsymbol{M})+|m_{ii}|h_j^S(\boldsymbol{M})}{(|m_{ii}|-h_i^S(\boldsymbol{M}))(|m_{jj}|-h_j^{\bar{S}}(\boldsymbol{M}))-h_i^{\bar{S}}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

2 Error bounds for linear complementarity problems of S-Nekrasov matrices

Theorem 1 Let $\boldsymbol{M}=(m_{ij})\in C^{n,n}$ be S-Nekrasov matrix, $\varnothing\neq S\subset N$, $m_{ii}>0$, $\forall i\in N$, $\tilde{\boldsymbol{M}}=\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M}=(\tilde{m}_{ij})$, $\boldsymbol{D}=diag(d_i)$, $0\le d_i\le 1$, then

$$\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M})^{-1}\|_\infty\le\max_{i\in N}\eta_i(\boldsymbol{M})\max_{i\in S,j\in\bar{S}}\max\left\{\rho_{ij}^S(\boldsymbol{M}),\rho_{ji}^{\bar{S}}(\boldsymbol{M})\right\}\tag{3}$$

$$\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M})^{-1}\|_\infty\le\max_{i\in N}\frac{\eta_i(\boldsymbol{M})}{\min\{m_{ii},1\}}\max_{i\in S,j\in\bar{S}}\max\left\{\tilde{\xi}_{ij}^S(\boldsymbol{M}),\tilde{\xi}_{ji}^{\bar{S}}(\boldsymbol{M})\right\}\tag{4}$$

$$\rho_{ij}^S(\boldsymbol{M})=\frac{\dfrac{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}{\min\{m_{jj}-h_j^{\bar{S}}(M),1\}}+\dfrac{h_j^S(\boldsymbol{M})(m_{ii}-h_i^S(\boldsymbol{M}))}{\min\{m_{ii}-h_i^S(M),1\}}}{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))-h_i^{\bar{S}}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

$$\rho_{ji}^{\bar{S}}(\boldsymbol{M})=\frac{\dfrac{(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))(m_{ii}-h_i^S(\boldsymbol{M}))}{\min\left\{m_{ii}-h_i^S(M),1\right\}}+\dfrac{h_i^{\bar{S}}(\boldsymbol{M})(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}{\min\left\{m_{jj}-h_j^{\bar{S}}(M),1\right\}}}{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))-h_i^{\bar{S}}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

$$\tilde{\xi}_{ij}^{S}(\boldsymbol{M})=\frac{(1+m_{jj})\dfrac{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}{\min\left\{m_{jj}-h_j^{\bar{S}}(M),1\right\}}+(1+m_{ii})\dfrac{h_j^S(\boldsymbol{M})(m_{ii}-h_i^S(\boldsymbol{M}))}{\min\left\{m_{ii}-h_i^S(\boldsymbol{M}),1\right\}}}{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))-h_i^{\bar{S}}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

$$\tilde{\xi}_{ji}^{\bar{S}}(\boldsymbol{M})=\frac{(1+m_{ii})\dfrac{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}{\min\left\{m_{ii}-h_i^S(\boldsymbol{M}),1\right\}}+(1+m_{jj})\dfrac{h_i^{\bar{S}}(\boldsymbol{M})(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}{\min\left\{m_{jj}-h_j^{\bar{S}}(M),1\right\}}}{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))-h_i^{\bar{S}}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

Proof: Let $\tilde{\boldsymbol{M}}=\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM}=(\tilde{m}_{ij})$, By Lemma 2, we can have $\tilde{\boldsymbol{M}}$ is S-Nekrasov matrix, then by Eq. 1of Lemma 5, we easily get

$$\left\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM})^{-1}\right\|_\infty\le\max_{i\in N}z_i(\tilde{\boldsymbol{M}})\max_{i\in S,j\in\bar{S}}\max\left\{\chi_{ij}^S(\tilde{\boldsymbol{M}}),\chi_{ji}^{\bar{S}}(\tilde{\boldsymbol{M}})\right\}$$

Application Lemma 2 and Lemma 3, when $\forall i\in S$, then

$$\frac{1}{\tilde{m}_{ii}-h_i^S(\tilde{\boldsymbol{M}})}=\frac{1}{1-d_i+m_{ii}d_i-(\sum\limits_{j=1}^{i-1}\left|\tilde{m}_{ij}\right|\dfrac{h_j^S(\tilde{\boldsymbol{M}})}{\left|\tilde{m}_{jj}\right|}+\sum\limits_{j=i+1,j\in S}^{n}\left|\tilde{m}_{ij}\right|)}$$

$$=\frac{1}{1-d_i+(m_{ii}-h_i^S(\boldsymbol{M}))d_i}\le\frac{1}{\min\left\{m_{ii}-h_i^S(\boldsymbol{M}),1\right\}}\qquad(5)$$

and, when $\forall j\in\bar{S}$

$$\frac{1}{\tilde{m}_{jj}-h_j^{\bar{S}}(\tilde{\boldsymbol{M}})}\le\frac{1}{\min\left\{m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}),1\right\}}\qquad(6)$$

Application Eq. 5, Eq.6 and Lemma 1, then

$$\chi_{ij}^S(\tilde{\boldsymbol{M}})=\frac{\left|\tilde{m}_{ii}\right|-h_i^S(\tilde{\boldsymbol{M}})+h_j^S(\tilde{\boldsymbol{M}})}{(\left|\tilde{m}_{ii}\right|-h_i^S(\tilde{\boldsymbol{M}}))(\left|\tilde{m}_{jj}\right|-h_j^{\bar{S}}(\tilde{\boldsymbol{M}}))-h_i^{\bar{S}}(\tilde{\boldsymbol{M}})h_j^S(\tilde{\boldsymbol{M}})}$$

$$\le\frac{\dfrac{1}{\min\left\{m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}),1\right\}}+\dfrac{d_jh_j^S(\boldsymbol{M})}{\min\left\{m_{ii}-h_i^S(\boldsymbol{M}),1\right\}d_j(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}}{1-\dfrac{d_ih_i^{\bar{S}}(\boldsymbol{M})d_jh_j^S(\boldsymbol{M})}{d_i(m_{ii}-h_i^S(\boldsymbol{M}))d_j(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}}$$

$$=\frac{\dfrac{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))}{\min\left\{m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}),1\right\}}+\dfrac{h_j^S(\boldsymbol{M})(m_{ii}-h_i^S(\boldsymbol{M}))}{\min\left\{m_{ii}-h_i^S(\boldsymbol{M}),1\right\}}}{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar{S}}(\boldsymbol{M}))-h_i^{\bar{S}}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

When $i\in S,j\in\bar{S}$, then

$$\rho_{ij}^{S}(\boldsymbol{M})=\frac{\dfrac{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar S}(\boldsymbol{M}))}{\min\left\{m_{jj}-h_j^{\bar S}(M),1\right\}}+\dfrac{h_j^S(\boldsymbol{M})(m_{ii}-h_i^S(\boldsymbol{M}))}{\min\left\{m_{ii}-h_i^S(M),1\right\}}}{(m_{ii}-h_i^S(\boldsymbol{M}))(m_{jj}-h_j^{\bar S}(\boldsymbol{M}))-h_i^{\bar S}(\boldsymbol{M})h_j^S(\boldsymbol{M})}$$

That is $\chi_{ij}^{S}(\tilde{\boldsymbol{M}})\le\rho_{ij}^{S}(\boldsymbol{M})$.

Similar to the proof Eq.3, we can get

$$\left\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM})^{-1}\right\|_\infty\le\max_{i\in N}\frac{\eta_i(\boldsymbol{M})}{\min\left\{m_{ii},1\right\}}\max_{i\in S,j\in\bar S}\max\left\{\tilde\xi_{ij}^{S}(\boldsymbol{M}),\tilde\xi_{ji}^{\bar S}(\boldsymbol{M})\right\}.$$

Numerical example

Let $\boldsymbol{M}_1=\begin{pmatrix}1&-\frac{3}{5}&-\frac{1}{5}&0\\-\frac{1}{4}&1&-\frac{1}{4}&-\frac{1}{2}\\-\frac{1}{5}&-\frac{2}{5}&1&-\frac{2}{5}\\-\frac{7}{9}&0&0&1\end{pmatrix}$, we easily verif, $\boldsymbol{M}_1$ is not Nekrasov matrix, but $\boldsymbol{M}_1$ is S-Nekrasov, $S=\{2,3\}$. By Eq.3, $\left\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM})^{-1}\right\|_\infty\le 76.715$.

By Theorem 5 of References[18], $\left\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM})^{-1}\right\|_\infty\le 110$.

3 Error bounds for linear complementarity problems of B-S-Nekrasov matrix

Let $\boldsymbol{M}=(m_{ij})\in R^{n,n}$, $\boldsymbol{M}=\boldsymbol{B}^{+}+\boldsymbol{C}$,

$$\boldsymbol{B}^{+}=(b_{ij})=\begin{pmatrix}m_{11}-r_1^{+}&\cdots&m_{1n}-r_1^{+}\\\vdots&&\vdots\\m_{n1}-r_n^{+}&\cdots&m_{nn}-r_n^{+}\end{pmatrix},\quad(7)\qquad \boldsymbol{C}=\begin{pmatrix}r_1^{+}&\cdots&r_1^{+}\\\vdots&&\vdots\\r_n^{+}&\cdots&r_n^{+}\end{pmatrix},\quad r_i^{+}=\max\left\{0,m_{ij}\,\middle|\,j\ne i\right\}$$

$\boldsymbol{B}^{+}$ is Z matrix, $\boldsymbol{C}$ is nonnegative matrix.

Lemma 6 Let real matrix $\boldsymbol{M}=(m_{ij})\in R^{n,n}$ be a B-S-Nekrasov matrix, with the form(7), and $\boldsymbol{B}^{+}$ is a S- Nekrasov matrix with all its principal minors are positive.

Theorem 2 Let $\boldsymbol{M}=(m_{ij})\in R^{n,n}$ be B-S- Nekrasov matrix, $\varnothing\ne\boldsymbol{S}\subset\boldsymbol{N}$, $\boldsymbol{B}^{+}$ with the form(7), then $\max_{d\in[0,1]^n}\left\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM})^{-1}\right\|_\infty\le(n-1)\max_{i\in N}\eta_i(\boldsymbol{B}^{+})\max_{i\in S,j\in\bar S}\max\left\{\rho_{ij}^{S}(\boldsymbol{B}^{+}),\rho_{ji}^{\bar S}(\boldsymbol{B}^{+})\right\}$.

Proof: because $\boldsymbol{M}$ is B-S- Nekrasov matrix, $\boldsymbol{M}=\boldsymbol{B}^{+}+\boldsymbol{C}$, $\boldsymbol{B}^{+}$ is a S- Nekrasov Z matrix with all its principal minors are positive, let $\boldsymbol{D}=diag(d_i)$, $0\le d_i\le 1$,

$$\tilde{\boldsymbol{M}}=\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM}=(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DB}^{+})+\boldsymbol{DC}=\boldsymbol{B}_{\boldsymbol{D}}^{+}+\tilde{\boldsymbol{C}}_{\boldsymbol{D}}\ \ \tilde{\boldsymbol{B}}_D^{+}=\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DB}^{+},\ \ \tilde{\boldsymbol{C}}_{\boldsymbol{D}}=\boldsymbol{DC}$$

By lemma 6, we can obtain that $\tilde{\boldsymbol{B}}_D^{+}$ is a $\tilde{\boldsymbol{B}}_D^{+}$ S- Nekrasov matrix with all its principal minors are positive, and $\tilde{\boldsymbol{B}}_D^{+}$ is non-singular M matrix, by the Theorem 2 of Reference [19], we can get, $\left\|M_D^{-1}\right\|_\infty\le\left\|(I+(\tilde B_D^{+})^{-1}\tilde C_D)^{-1}\right\|_\infty\left\|(\tilde{\boldsymbol{B}}_D^{+})^{-1}\right\|_\infty\le(n-1)\left\|(\tilde{\boldsymbol{B}}_D^{+})^{-1}\right\|_\infty$

Because $\tilde{\boldsymbol{B}}_D^{+}$ is S- Nekrasov matrix, by Theorem 1, we can get

$$\left\|(\tilde{\boldsymbol{B}}_D^{+})^{-1}\right\|_\infty\le\max_{i\in S,j\in\bar S}\max\left\{\chi_{ij}^{S}(\tilde{\boldsymbol{B}}_D^{+}),\chi_{ji}^{\bar S}(\tilde{\boldsymbol{B}}_D^{+})\right\}\le\max_{i\in S,j\in\bar S}\max\left\{\rho_{ij}^{S}(\tilde{\boldsymbol{B}}_D^{+}),\rho_{ji}^{\bar S}(\tilde{\boldsymbol{B}}_D^{+})\right\},$$

Then $\left\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM})^{-1}\right\|_{\infty} \leq (n-1)\max_{i\in N}\eta_i(\boldsymbol{B}^+)\max_{i\in S, j\in \bar{S}}\max\left\{\rho_{ij}^{S}(\boldsymbol{B}^+), \rho_{ji}^{\bar{S}}(\boldsymbol{B}^+)\right\}$ $_{d\in[0,1]^n}$

Theorem 3 let $\boldsymbol{M}=(m_{ij})\in R^{n,n}$ be B-S- Nekrasov matrix, $\varnothing \neq \boldsymbol{S} \subset \boldsymbol{N}$, $\boldsymbol{B}^+$ with the form(7), then $\max_{d\in[0,1]^n}\left\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{DM})^{-1}\right\|_{\infty} \leq (n-1)\max_{i\in N}\frac{\eta_i(\boldsymbol{B}^+)}{\min\left\{b_{ii}^+,1\right\}}\max_{i\in S, j\in \bar{S}}\max\left\{\tilde{\xi}_{ij}^{S}(\boldsymbol{B}^+), \tilde{\xi}_{ji}^{\bar{S}}(\boldsymbol{B}^+)\right\}$.

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